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Duality for a nonlinear fractional programming under fuzzy environment with parabolic concave membership functions

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ABSTRACT

A particular type of convex fractional programming problem and its dual is studied under fuzzy environment with parabolic concave membership functions. Appropriate duality results are established using aspiration level approach. The use of parabolic concave membership functions to represent the degree of satisfaction of the decision maker makes it unique from the other studies.

1. Introduction

Mathematical programming finds many applications in the field of management. Optimization of resources in any organization is very much handled by the application of mathematical programming. An important class of mathematical programming problems is fractional programming which deals with situations where a ratio between two mathematical functions is either maximized or minimized.

There are many managerial decision making situations where the uncertainties in working situations is best explained by fuzzy set theory. The concept of fuzzy set theory is introduced by Zadeh (1965), since then a large number of researchers have shown their interest in the application of fuzzy set theory. Bellman and Zadeh (1970) proposed the concept of decision making in fuzzy environment and their concept of fuzzy decision making is used by Tanaka et al. (1984) in mathematical programming. There are many authors have discussed the use of fuzzy set theory in fractional programming e.g., Luhandjula (1984), Dutta et al. (1992), Ravi and Reddy (1998), Gupta and Bhatia (2001), Chakraborty and Gupta (2002), Pop and Stancu-Minasian (2003), and Stancu-Minasian and Pop (2008).

The duality theory plays a very important role in the theory of linear programming so researchers have shown their interest in the concept of duality for a linear program under fuzzy environment as well e.g., Hamacher et al. (1978), Rödder and Zimmermann (1980), Bector and Chandra (2002) and few others. However, only a few studies exploring duality in fractional programming under fuzzy environment are available in literature. Lee et al. (1991) studied duality for a fuzzy multiobjective linear fractional programming problem and developed a parallel algorithm. Wu (2007) developed duality theory in fuzzy optimization problems formulated by the Wolfe's primal and dual pair. Gupta and Mehlawat (2009a) studied duality for a convex fractional programming under fuzzy environment using linear membership functions.

It is important to note that while implementing any fuzzy mathematical programming problem on the basis of aspiration levels the choice of membership function is very important. The chosen membership function should be able to produce desired satisfaction level of the objective of the decision maker. Several membership functions have been employed in fuzzy mathematical programming: (i) linear (Zimmermann, 2001) (ii) piecewise linear (Inuiguchi et al., 1990) (iii) parabolic concave (Saxena and Jain, 2014) (iv) exponential (Gupta and Mehlawat, 2009b; Li and Lee, 1991). In many practical situations, however, a linear membership function is not a suitable representation which is empirically shown by Hersh and Caramazza (1976). A nonlinear membership function can be used to obtain the desired degree of satisfaction of the objective of the decision maker. However it must be noted that the results obtained for a fuzzy environment must conform to the corresponding results for the crisp situation.

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In this paper, we attempt to obtain duality results between a particular type of convex fractional programming problem and its dual under fuzzy environment using a nonlinear membership function i.e., parabolic concave membership function. The use of parabolic concave membership functions to obtain the desired satisfaction level of the decision maker's objective makes it unique study in this direction. The duality results obtained under fuzzy environment are also conforming to the corresponding duality results for the crisp situation. The economic interpretation of these results can be understood as explained by Rödder and Zimmermann (1980).

The paper is organized into six sections. Section 2 contains notation and prerequisites. In section 3 a pair of fuzzy primal and dual problems for convex fractional programming is presented. In section 4 a modified weak duality theorem and some other related results are proved. In section 5 a numerical example is presented to verify the results established in section 4. A conclusion is presented in the final section 6.

2. Notation and prerequisites

Let R^n denote the n-dimensional Euclidean space and R_+^n be its non-negative orthant. Consider the following convex fractional programming problem and its dual as studied in (Stancu-Minasian, 1997)(Eqs.1-4):

(P) minimize $f(x) = \frac{(c^t x)^2}{d^t x}$ (1)

subject to $Ax \geq b, x \geq 0$ (2)

(D) maximize $g(u,v) = b^t u$ (3)

subject to $A^t u + dv^2 \leq 2cv, u, v \geq 0$ (4)

where, the vectors $x \in R^n, d \in R^n, c \in R^n$ and $\in R^m, u \in R^m, v \in R, A \in R^{m\times n}$

Let $S = \{x \mid Ax \geq b, x \geq 0\}$ be the set of feasible solutions for the primal problem (P). Assuming $c^t x \geq 0$ and $d^t x > 0$ on S. Let $T = \{(u,v) \mid A^t u + dv^2 \leq 2cv, u \geq 0, v \geq 0\}$ be the set of feasible solutions for the dual problem (D).

3. Fuzzy primal-dual convex

3.1. Fractional programming problems

Now consider the fuzzy versions $(\tilde{P})$ and $(\tilde{D})$ of (P) and (D) respectively, in the sense of Rödder and Zimmermann (1980).

$(\tilde{P})$ Find $x \in R^n$ such that (Eqs. 5 and 6):

$f(x) = \frac{(c^t x)^2}{d^t x} \underset{\sim}{<} Z_0,$ (5)

$Ax \underset{\sim}{>} b, x \geq 0.$ (6)

$(\tilde{D})$ Find $u \in R^m, v \in R$, such that (Eqs. 7 and 8):

$g(u,v) = b^t u \underset{\sim}{>} W_0$ (7)

$A^t u + dv^2 \underset{\sim}{<} 2cv, u, v \geq 0$ (8)

Here, "$\underset{\sim}{>}$" and "$\underset{\sim}{<}$" are fuzzy versions of symbols "$\geq$" and "$\leq$" respectively, and have the linguistic interpretation "essentially greater than or equal" and "essentially less than or equal" as explained in Rödder and Zimmermann (1980). These indicate that the inequalities are flexible and may be described by a fuzzy set whose membership function represents fulfillment of the decision maker's satisfaction. Also Z_0 and W_0 are aspiration level of the two objectives.

We now assume $p_0 > 0, p_i > 0, (i = 1,2,..., m)$ as subjectively chosen constants of admissible violations such that p_0 is associated with the objective function and p_i $(i = 1,2,..., m)$ is associated with the i-th linear constraint of (P).

Now we define parabolic concave membership functions $\mu_0^P(f(x)): R \to [0,1]$ and $\mu_i^P(A_i x): R \to [0,1]$, $(i = 1,2, \dots, m)$ for objective function and constraints of the problem $(\tilde{P})$ to obtain a degree of satisfaction in the problem:

$$\mu_0^P(f(x)) = \begin{cases} 1, & \text{if } f(x) \leq Z_0 \\ 1 - \left(\frac{f(x) - Z_0}{p_0}\right)^2, & \text{if } Z_0 < f(x) \leq Z_0 + p_0 \\ 0, & \text{if } f(x) > Z_0 + p_0 \end{cases}$$

and

$$\mu_i^P(A_i x) = \begin{cases} 1, & \text{if } A_i x \geq b_i \\ 1 - \left(\frac{b_i - A_i x}{p_i}\right)^2, & \text{if } b_i - p_i \leq A_i x < b_i \\ 0, & \text{if } A_i x < b_i - p_i \end{cases} \quad i = 1,2, \dots, m$$

Using the "min" operator to aggregate the overall satisfaction and following Rödder and Zimmermann (1980) with these membership functions, the crisp equivalent of the fuzzy primal convex fractional programming problem $(\tilde{P})$ is as follow:

(CP) minimize $(-\lambda)$

subject to $\lambda \leq 1 - \left(\frac{f(x) - Z_0}{p_0}\right)^2$

$\lambda \leq 1 - \left(\frac{b_i - A_i x}{p_i}\right)^2 \quad i = 1,2, \dots, m$

$\lambda \leq 1$

$x, \lambda \geq 0$

where, A_i $(i = 1,2, \dots, m)$ denotes the i-th row of the matrix A and b_i $(i = 1,2, \dots, m)$ denotes the i-th component of vector b.

Similarly, assume that $q_j > 0, (j = 0,1,2,..., n)$ are subjectively chosen constants of admissible violations of the objective and the constraints of (D). Now we define parabolic concave membership functions $\mu_0^D(.)$ and $\mu_j^D(.) (j = 1,2,..., n)$ for objective function and constraints of the problem $(\tilde{D})$:

$$\mu_0^D(.) = \begin{cases} 1, & \text{if } g(u,v) \geq W_0 \\ 1 - \left(\frac{W_0 - g(u,v)}{q_0}\right)^2, & \text{if } W_0 - q_0 \leq g(u,v) < W_0 \\ 0, & \text{if } g(u,v) < W_0 - q_0 \end{cases}$$

and

$$\mu_j^D(.) = \begin{cases} 1 & ,\text{if } A_j^t u + d_j v^2 - 2c_j v \le 0 \\ 1 - \left(\dfrac{A_j^t u + d_j v^2 - 2c_j v}{q_j}\right)^2 & ,\text{if } 0 < A_j^t u + d_j v^2 - 2c_j v \le q_j \; j = 1,2,..., \\ 0 & ,\text{if } A_j^t u + d_j v^2 - 2c_j v > q_j \end{cases}$$

Using above membership functions, the crisp equivalent of fuzzy dual convex fractional programming problem $(\widetilde{D})$ is as follows:

(CD) maximize η

subject to $\eta \le 1 - \left(\dfrac{W_0 - g(u,v)}{q_0}\right)^2$

$$\eta \le 1 - \left(\frac{A_j^t u + d_j v^2 - 2c_j v}{q_j}\right)^2 \quad j = 1,2, \dots, n$$

$$\eta \le 1$$

$$u, v, \eta \ge 0$$

where, A_j^t $(j = 1,2,...,n)$ denote the j-th row of the matrix $A^t$and $d_j$and c_j $(j = 1,2,...,n)$denotes the j-th components of d and c respectively.

The problems (CP) and (CD) can be alternatively rewritten as (EP) and (ED) respectively.

(EP) minimize $(-\lambda)$

subject to $f(x) - Z_0 \le \sqrt{1-\lambda}p_0$

$$b_i - A_i x \le \sqrt{1-\lambda}p_i \; i = 1,2, \dots, m$$

$$\lambda \le 1$$

$$x, \lambda \ge 0$$

(ED) maximize η

subject to $W_0 - g(u,v) \le \sqrt{1-\eta}q_0$

$$A_j^t u + d_j v^2 - 2c_j v \le \sqrt{1-\eta}q_j \quad j = 1,2, \dots, n$$

$$\eta \le 1$$

$$u, v, \eta \ge 0$$

We name the pair (EP)-(ED) as the modified primal-dual pair of fuzzy convex fractional programming problems.

4. Modified weak duality

4.1. Theorem and related results

Now we establish appropriate duality results for the modified primal-dual pair (EP)-(ED) (or equivalently (CP)-(CD)).

Theorem 1: (Modified Weak Duality). Let (x, λ)be (EP)-feasible and (u, v, η) be (ED)-feasible. Then,

$$g(u,v) - f(x) \le \sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} + \sum_{i=1}^{m} q_j x_j \sqrt{1-\eta}$$

or,

$$g(u,v) - f(x) \le \sqrt{1-\lambda}p^t u + \sqrt{1-\eta}q^t x$$

Proof: Assume that $S \ne \varphi$ and $T \ne \varphi$. Since $(x, \lambda)$is (EP)-feasible, therefore we have

$$(b_i - A_i x) \le \sqrt{1-\lambda}p_i, i = 1,2, \dots, m \quad (9)$$

since (u, v, η) is (ED)-feasible, we have

$$A_j^t u + d_j v^2 - 2c_j v \le \sqrt{1-\eta}q_j, \; j = 1,2, \dots, n \quad (10)$$

multiplying (9) by u_i, $(\, i = 1,2, \dots, m)$

$$(b^t u - x^t A^t u) \le \textstyle\sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} \quad (11)$$

multiplying (10) by x_j, $(j = 1,2, \dots, n)$

$$(x^t A^t u + x^t \mathrm{d} v^2 - 2x^t \mathrm{c} v) \le \textstyle\sum_{j=1}^{n} q_j x_j \sqrt{1-\eta} \quad (12)$$

adding (11) and (12), we get

$$x^t \mathrm{d} v^2 - 2x^t \mathrm{c} v + b^t u \le \sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} + \sum_{j=1}^{n} q_j x_j \sqrt{1-\eta}$$

$$\Rightarrow b^t u + (v\sqrt{d^t x} - c^t x/\sqrt{d^t x})^2 - \frac{(c^t x)^2}{d^t x} \le \sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} + \sum_{j=1}^{n} q_j x_j \sqrt{1-\eta}$$

since $(v\sqrt{d^t x} - c^t x/\sqrt{d^t x})^2 > 0$

$$b^t u - \frac{(c^t x)^2}{d^t x} \le \sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} + \sum_{j=1}^{n} q_j x_j \sqrt{1-\eta}$$

$$\Rightarrow g(u,v) - f(x) \le \textstyle\sum_{i=1}^{m} p_i u_i \sqrt{1-\lambda} + \sum_{j=1}^{n} q_j x_j \sqrt{1-\eta} \quad (13)$$

or,

$$g(u,v) - f(x) \le \sqrt{1-\lambda}p^t u + \sqrt{1-\eta}q^t x$$

This proves the result.

Remark 1: It may be noted that for $\lambda = 1$ and $\eta = 1$inequality (13) reduces to$g(u,v) \le f(x)$, which is the standard weak duality result in the crisp duality theory. Also, for $0 < \lambda < 1$and $0 < \eta < 1$ the situation remains fuzzy which, for the given tolerance levels p and q, is quantified in the expression

$$\sqrt{1-\lambda}p^t u + \sqrt{1-\eta}q^t x$$

Remark 2: In addition to inequality (13), using

$$f(x) - Z_0 \le \sqrt{1-\lambda}p_0$$

and

$$W_0 - g(u,v) \le \sqrt{1-\eta}q_0,$$

it can also be proved that

$$f(x) - g(u,v) + (W_0 - Z_0) \le \sqrt{1-\lambda}p_0 + \sqrt{1-\eta}q_0 \quad (14)$$

this inequality relates the relative difference of the aspiration level Z_0 of $f(x)$ and W_0 of $g(u,v)$ respectively, in terms of their tolerance levels

p_0 and q_0. When $\lambda = 1$ and $\eta = 1$; then (13) yields $g(u,v) \le f(x)$ and in turn (14) implies $W_0 \le Z_0$.

Corollary 1: Let $(\hat{x},\hat{\lambda})$ be (EP)-feasible and $(\hat{u},\hat{v},\hat{\eta})$ be (ED)-feasible such that

(i) $\sqrt{1-\hat{\eta}}q^t\hat{x} + \sqrt{1-\hat{\lambda}}p^t\hat{u} = g(\hat{u},\hat{v}) - f(\hat{x})$

(ii) $q_0\sqrt{1-\hat{\eta}} + p_0\sqrt{1-\hat{\lambda}} = (f(\hat{x}) - g(\hat{u},\hat{v})) + (W_0 - Z_0)$

(iii) $(W_0 - Z_0) \le 0$

then, the following results hold:

(a) $(\hat{x},\hat{\lambda})$ is (EP)-optimal and $(\hat{u},\hat{v},\hat{\eta})$ is (ED)-optimal.

(b) $(\hat{x},\hat{\lambda},\hat{u},\hat{v},\hat{\eta})$ is an optimal solution to the following problem (MP) whose maximum objective value is zero.

(MP) max $[g(u,v) - f(x) - \sqrt{1-\eta}q^t x - \sqrt{1-\lambda}p^t u]$

subject to $f(x) - Z_0 \le \sqrt{1-\lambda}p_0$

$g(u,v) - W_0 \le \sqrt{1-\eta}q_0$

$(b_i - A_i x) \le \sqrt{1-\lambda}p_i, i = 1,2,\dots,m$

$(A_j^t u + d_j v^2) - 2c_j v \le \sqrt{1-\eta}q_j, j = 1,2,\dots,n$

$\lambda \le 1$

$\eta \le 1$

$x,\lambda,u,v,\eta \ge 0$

Proof: Let (x,λ) be (EP)-feasible and (u,v,η) be (ED)-feasible then by modified weak duality theorem

$$g(u,v) - f(x) - \sqrt{1-\eta}q^t x - \sqrt{1-\lambda}p^t u \le 0 \quad (15)$$

from (i) we have

$$g(\hat{u},\hat{v}) - f(\hat{x}) - \sqrt{1-\hat{\eta}}q^t\hat{x} - \sqrt{1-\hat{\lambda}}p^t\hat{u} = 0 \quad (16)$$

(15) and (16) gives

$$g(u,v) - f(x) - \sqrt{1-\eta}q^t x - \sqrt{1-\lambda}p^t u \le g(\hat{u},\hat{v}) - f(\hat{x}) - \sqrt{1-\hat{\eta}}q^t\hat{x} - \sqrt{1-\hat{\lambda}}p^t\hat{u} \quad (17)$$

this implies that $(\hat{x},\hat{\lambda},\hat{u},\hat{v},\hat{\eta})$ is optimal to (MP).

From (i), we have

$$g(\hat{u},\hat{v}) - f(\hat{x}) - \sqrt{1-\hat{\eta}}q^t\hat{x} - \sqrt{1-\hat{\lambda}}p^t\hat{u} = 0 \quad (18)$$

from (ii), we have

$$-g(\hat{u},\hat{v}) + f(\hat{x}) + (W_0 - Z_0) - q_0\sqrt{1-\hat{\eta}} - p_0\sqrt{1-\hat{\lambda}} = 0 \quad (19)$$

adding (18) and (19), we get

$$-\sqrt{1-\hat{\eta}}q^t\hat{x} - \sqrt{1-\hat{\lambda}}p^t\hat{u} - p_0\sqrt{1-\hat{\lambda}} - q_0\sqrt{1-\hat{\eta}} + (W_0 - Z_0) = 0 \quad (20)$$

now, each term of (20) is non-positive, therefore

$$-\sum_{i=1}^{m}\sqrt{1-\hat{\lambda}}p_i\hat{u}_i = 0, -\sum_{j=1}^{n}\sqrt{1-\hat{\eta}}q_j\hat{x}_j = 0,$$
$$-p_0\sqrt{1-\hat{\lambda}} = 0, -q_0\sqrt{1-\hat{\eta}} = 0, \quad (21)$$

and

$$(W_0 - Z_0) = 0$$

since, $p_0 > 0$, $q_0 > 0$ and $\lambda \le 1$, $\eta \le 1$, therefore

$$-p_0\sqrt{1-\lambda} \le 0 \text{ and } -q_0\sqrt{1-\eta} \le 0 \quad (22)$$

from (21) and (22) we have

$-p_0\sqrt{1-\lambda} \le -p_0\sqrt{1-\hat{\lambda}}$ and $-q_0\sqrt{1-\eta} \le -q_0\sqrt{1-\hat{\eta}}$

$\Rightarrow \sqrt{1-\lambda} \ge \sqrt{1-\hat{\lambda}}$ and $\sqrt{1-\eta} \ge \sqrt{1-\hat{\eta}}$

$\Rightarrow \lambda \le \hat{\lambda}$ and $\eta \le \hat{\eta}$

or,

$-\hat{\lambda} \le -\lambda$ and $\eta \le \hat{\eta}$

This proves the result.

Remark 3: Since, (CD) is not a dual to (CP) in the conventional sense, however (CP) and (CD) are the crisp equivalent of the fuzzy pair $(\tilde{P})$ and $(\tilde{D})$ respectively, therefore, there does not exist any strong duality theorem between them. However, in addition to $\lambda = 1$ and $\eta = 1$, we also have $Z_0 - W_0 = 0$, then inequalities (13) and (14) yields $g(u,v) = f(x)$ i.e., x and (u,v) become optimal solution to the problems (P) and (D) respectively.

5. Numerical example

In this section we present a simple numerical example to illustrate the construction of the fuzzy primal-dual pair and also to verify the modified weak duality theorem.

Let us consider the following pair of primal-dual convex fractional programming problem:

(P) minimize $f(x) = \frac{(2x_1+x_2)^2}{x_1+2x_2}$

subject to

$2x_1 + x_2 \ge 6,$

$x_1 + 3x_2 \ge 8,$

$x_1, x_2 \ge 0$

(P)-optimal is $x_1^0 = 0, x_2^0 = 6.0$, and $f(x^0) = 3.0$

(D) maximize $g(u,v) = 6u_1 + 8u_2$

subject to

$2u_1 + u_2 + v^2 - 4v \le 0$

$u_1 + 3u_2 + 2v^2 - 2v \le 0$

$u_1, u_2, v \ge 0$

(D)-optimal is $u_1^0 = 0.5, u_2^0 = 0.1978609E-08, v^* = 5.0$, and $g(u^0,v^0) = 3.0$.

Consider the fuzzified version $(\tilde{P})$ of (P) and taking $p_0 = 2, p_1 = 1, p_2 = 2$, and $Z_0 = 1$, with respect to $(\tilde{P})$, the corresponding (EP) becomes

(EP) minimize $(-\lambda)$

subject to

$4x_1^2 + 4x_1x_2 + x_2^2 - 2\sqrt{1-\lambda}.x_1 - 4\sqrt{1-\lambda}.x_2 - x_1 - 2x_2 \le 0$

$2x_1 + x_2 + \sqrt{1-\lambda} \geq 6$
$x_1 + 3x_2 + 2\sqrt{1-\lambda} \geq 8$
$\lambda \leq 1$
$x_1, x_2, \lambda \geq 0$

The optimal solution of (EP) is at $x_1^* = 0$, $x_2^* = 5.2, \lambda^* = 0.36$ and the optimal value of (EP)-objective is $-\lambda^* = -0.36$.

Similarly, considering the fuzzified version $(\widetilde{D})$ of (D) and taking $q_0 = 1, q_1 = 1, q_2 = 2$, and $W_0 = 1$ with respect to $(\widetilde{D})$, the corresponding problem (ED) becomes

(ED) maximize η
subject to
$6u_1 + 8u_2 + \sqrt{1-\eta} \geq 1$
$2u_1 + u_2 + v^2 - 4v - \sqrt{1-\eta} \leq 0$
$u_1 + 3u_2 + 2v^2 - 2v - 2\sqrt{1-\eta} \leq 0$
$\eta \leq 1$
$u_1, u_2, v, \eta \geq 0$

The optimal solution of (ED) is at $\eta^* = 1, u_1^* = 0.39, u_2^* = 0, v^* = 0.74$ and the optimal value of (ED)-objective is $\eta^* = 1$. For these optimal solutions both the inequalities (13) and (14) are satisfied.

6. Conclusion

In this paper, along the lines of Gupta and Mehlawat (2009a), a pair of primal and dual programs for a convex fractional program under fuzzy environment is presented with parabolic concave membership functions. We consider a conventional primal-dual pair as (P) and (D) and obtain their fuzzified versions $(\widetilde{P})$ and $(\widetilde{D})$ using Rödder and Zimmermann (1980) approach. Next, using Saxena and Jain (2014) and Rödder and Zimmermann (1980), the crisp formulations of $(\widetilde{P})$ and $(\widetilde{D})$ are obtained as (EP) and (ED) (or equivalently (CP) and (CD)) respectively. A modified weak duality theorem relating feasible solution of (EP) and (ED) is proved and a corollary is also proved relating optimal solutions of (EP) and (ED). The crisp equivalents (EP) and (ED) obtained are nonlinear problems, where non linearity exists in the constraints. Software LINGO (Scharge, 1997) has been used to solve the numerical illustration. Fuzzy decisive set method (Sakawa and Yano, 1985) and the modified subgradient method (Gasimov, 2002) can also be used to solve these problems. Duals of other types can also be examined under fuzzy environment for the fractional programming problem under consideration for similar kinds of results as obtained in this paper. The approach developed here will prove helpful for possible extensions to linear fractional and quadratic fractional programs and to various other nonlinear fractional programming problems under fuzzy environment with parabolic concave membership functions. Other nonlinear membership functions such as hyperbolic, exponential etc. can also be employed, provided it should conform to the corresponding duality results for the crisp situation.

References

Bector CR and Chandra S (2002). On duality in linear programming under fuzzy environment. Fuzzy Sets and Systems 125(3): 317-325

Bellman RE and Zadeh LA (1970). Decision-making in a fuzzy environment. Management Science, 17(4): 141-164.

Chakraborty M and Gupta S (2002). Fuzzy mathematical programming for multi objective linear fractional programming problem. Fuzzy sets and systems, 125(3): 335-342.

Dutta D, Tiwari RN, and Rao JR (1992). Multiple objective linear fractional programming—a fuzzy set theoretic approach. Fuzzy Sets and Systems, 52(1): 39-45.

Gasimov RN (2002). Augmented Lagrangian duality and no differentiable optimization methods in no convex programming. Journal of Global Optimization, 24(2): 187-203.

Gupta P and Bhatia D (2001). Sensitivity analysis in fuzzy multiobjective linear fractional programming problem. Fuzzy Sets and Systems, 122(2): 229-236.

Gupta P and Mehlawat MK (2009a). Duality for a convex fractional programming under fuzzy environment. International Journal of Optimization: Theory, Methods and Applications, 1(3): 291-301.

Gupta P and Mehlawat MK (2009b). Bector–Chandra type duality in fuzzy linear programming with exponential membership functions. Fuzzy Sets and Systems, 160(22): 3290-3308.

Hamacher H, Leberling H, and Zimmermann HJ (1978). Sensitivity analysis in fuzzy linear programming. Fuzzy Sets and Systems, 1(4): 269-281.

Hersh HM and Caramazza A (1976). A fuzzy set approach to modifiers and vagueness in natural language. Journal of Experimental Psychology: General, 105(3): 254. -276.

Inuiguchi M, Ichihashi H, and Kume Y (1990). A solution algorithm for fuzzy linear programming with piecewise linear membership functions. Fuzzy Sets and Systems, 34(1): 15-31.

Lee BI, Chung NK, and Tcha DW (1991). A parallel algorithm and duality for a fuzzy multiobjective lienar fractional programming problem. Computers and industrial engineering, 20(3): 367-372.

Li RJ and Lee ES (1991). An exponential membership function for fuzzy multiple objective linear programming. Computers and Mathematics with Applications, 22(12): 55-60.

Luhandjula MK (1984). Fuzzy approaches for multiple objective linear fractional optimizations. Fuzzy Sets and Systems, 13(1): 11-23.

Pop B and Stancu-Minasian IM (2003). On a fuzzy set approach to solving multiple objective linear fractional programming problems. Fuzzy Sets and Systems, 134(3): 397-405.

Ravi V and Reddy PJ (1998). Fuzzy linear fractional goal programming applied to refinery operations planning. Fuzzy Sets and Systems, 96(2): 173-182.

Rödder W and Zimmermann HJ (1980). Duality in fuzzy linear programming. In: Fiacco AV and Kortanek KO (Eds.), Extremal methods and system analysis: 415–429. Springer, Berlin/New York. https://doi.org/10.1007/978-3-642-46414-0_20

Sakawa M and Yano H (1985). Interactive decision making for multi-objective linear fractional programming problems with fuzzy parameters. Cybernetics and System, 16(4): 377-394.

Saxena P and Jain R (2014). Bector-Chandra type linear programming duality under fuzzy environment with parabolic concave membership functions. In 3rd International Conference on Reliability, Infocom Technologies and Optimization (ICRITO) (Trends and Future Directions), IEEE, Noida, India: 1-6. https://doi.org/10.1109/ICRITO.2014.7014726

Scharge L (1997). Optimization modeling with LINDO. Duxbury Press, California, USA.

Stancu-Minasian IM (1997). Fractional programming: Theory, methods and applications. Kluwer Academic Publishers, Dordrecht, Netherlands.

Stancu-Minasian IM and Pop B (2008). A method of solving fully fuzzified linear fractional programming problems. Journal of Applied Mathematics and Computing, 27(1-2): 227-242.

Tanaka H, Okuda T, and Asai K (1984). On fuzzy mathematical programming. Journal of Cybernetics, 3(4): 37-46.

Wu HC (2007). Duality theory in fuzzy optimization problems formulated by the Wolfe's primal and dual pair. Fuzzy Optimization and Decision Making, 6(3): 179-198.

Zadeh LA (1965). Fuzzy sets. Information and Control, 8(3): 338-353.

Zimmermann HJ (2001). Fuzzy set theory and its applications. 4th Edition, Kluwer Academic Publishers, Dordrecht, Netherlands.